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1991 J. Phys. A: Math. Gen. 24 735

(http://iopscience.iop.org/0305-4470/24/3/031)

J. Phys. A: Math. Gen. **24** (1991) 735–740. Printed in the UK

COMMENT

Random walk in a percolation cluster: external field dependence

Jae Woo Lee, Ho Chul Kim and Jong-Jean Kim†

Physics Department, Korea Advanced Institute of Science and Technology, PO Box 150, Chongyangni, Seoul, Korea

Received 3 September 1990

Abstract. Random walk in a percolation cluster was studied in the simultaneous application of both DC bias and AC driving fields by Monte Carlo calculations. Amplitude and phase shift of the random walk response to the external field of $B_0+B_1 \sin \omega t$ were calculated as a function of B_1, ω and B_0. Some new observations are made, which cannot be expected simply from the linear superposition of the two separate results for a constant bias field (B_0) and an AC driving field ($B_1 \sin \omega t$) respectively.

1. Introduction

Diffusions in the percolation clusters under constant bias fields have been studied by many research workers both theoretically (Barma and Dhar 1983, Ohtsuki and Keyes 1984, White and Barma 1984, Goldhirsch and Gefen 1987) and numerically (Pandey 1984, Seifert and Suessenbach 1984, Stauffer 1985, Bunde *et al* 1987).

With a constant field applied on the percolation cluster the random walk particle was observed to show a diffusion-like dynamics at short times but a drift-like one at long times when the bias field was kept smaller than a characteristic value B_c (Pandey 1984, Stauffer 1985).

When the bias field becomes time-dependent the random walk particle in the percolation cluster shows a nonlinear response, which is attributed to competition between two opposing effects of drift and trapping (Harder *et al* 1986, Havlin and Ben-Avraham 1987).

Trapping of the random walk particle at dangling bonds tends to reduce the root-mean-square displacement $\langle x(t)\rangle$ and counteracts the drifting of the particle given by the applied field.

In our present work we want to study the DC bias field effect on the nonlinear behaviours of the random walk responses in the percolation cluster under the time-dependent applied field.

2. Random walk in a percolation cluster

In the uniform regular lattice structures the root-mean-square displacement $\langle R(t)\rangle$ of the random walk particle under the time-dependent field of the form $B(t)=B_1 \sin \omega t$ is given by (Harder *et al* 1986)

$$\langle R(t)\rangle \sim A\{\sin(\omega t-\phi)+\Delta\} \tag{1}$$

† Author to whom correspondence should be addressed.

where $A(\sim B_1/\omega)$ represents the response amplitude, $\phi(=\pi/2)$ the phase lag and $\Delta(=1)$ the field-independent background. A random walk in the percolation cluster is characterized by the anomalous diffusion described as (Ben-Avraham and Havlin 1982, Gefen *et al* 1983)

$$\langle R^2(t)\rangle \sim t^{2k} \tag{2}$$

where $\langle R^2\rangle$ is the mean-square displacement with $k \leqslant \frac{1}{2}$.

The mean-square displacement $\langle R^2(t)\rangle$ becomes dependent on the field when a constant field is applied in the percolation cluster (Pandey 1984, Stauffer 1985).

Conductivity $\sigma(\omega)$ of the random network is given by (Scher and Lax 1973)

$$\sigma(\omega) \sim -\omega^2 \lim_{\eta\to 0} \int_0^\infty \exp(-\mathrm{i}\omega t)\exp(-\eta t)\langle R^2(t)\rangle \,\mathrm{d}t \tag{3}$$

where $\langle R^2(t)\rangle$ is the mean-square displacement of the random walk particle.

We thus have from (2) and (3)

$$\sigma(\omega) \sim \exp\left\{\mathrm{i}(1-2k)\frac{\pi}{2}\right\}\omega^{(1-2k)}. \tag{4}$$

From the Fourier transform relations of $j(\omega') \sim \sigma(\omega')B(\omega')$, $B(\omega') = B_1\delta(\omega-\omega')$ and also $j(t) = \mathrm{d}\langle x(t)\rangle/\mathrm{d}t$ the linear response theory gives (Harder *et al* 1986) for the response amplitude $A(B_1, \omega)$ and phase shift ϕ:

$$A(B_1, \omega) \sim B_1\omega^{-2k} \tag{5}$$

$$\phi = k\pi. \tag{6}$$

Although both DC bias (Pandey 1984, Stauffer 1985) and AC field (Harder *et al* 1986, Havlin and Ben-Avraham 1987) effects on the random walk have been studied separately, it is not obvious to see the random walk dynamics in the case of simultaneous application of both DC and AC fields since the DC field is expected to change both drifting and trapping dynamics. What is more, a strong AC field may also give rise to the nonlinear dynamic drive of the random walk particle in the percolation cluster, when a slightest possible change of initial conditions may give a drastic change in the response.

3. Monte Carlo results and discussion

A Leath algorithm (Leath 1976) was used to generate a two-dimensional percolation cluster of $p = 0.594$ up to 100 shells. Both DC and time-dependent fields are applied simultaneously in the form of $B(t) = B_0 + B_1 \sin \omega t$ with the field direction along the xy diagonal. Monte Carlo methods are employed to calculate the root-mean-square displacement $\langle x(t)\rangle$ and the response amplitudes A from the peak to valley amplitudes of $\langle x(t)\rangle$. For each percolation cluster generated we have taken 500 local-origin averages and 10 configuration averages. The transition rate W between neighbouring sites r and $r'(=r+\delta)$ was taken as

$$W_{r,r+\delta} = \begin{cases} (1+B(t))/4 & \text{for } \delta = (1,0) \text{ or } (0,1) \\ (1-B(t))/4 & \text{for } \delta = (-1,0) \text{ or } (0,-1) \end{cases} .$$

with $|B(t)| \leqslant 1$.

With no constant bias field, $B_0 = 0$, and only the time-dependent field applied we could reproduce the same results of the root-mean-square displacement $\langle x(t)\rangle$ as those of Harder *et al* (1986) which show the nonlinear effects with respect to the amplitude B_1 and frequency ω of the applied field. In figure 1 we have shown the $\langle x(t)\rangle$ dependence on t at various B_1 values for fixed values of $B_0=0.1$ and $\omega=0.02$. At $B_1=0$ $\langle x(t)\rangle$ is seen to reproduce the result of the DC bias field in agreement with the previous results (Pandey 1984, Stauffer 1985). At $B_1=B_0=0.1$ we can observe a strong effect of the DC bias field. However, as the AC field becomes much stronger the DC bias effect appears suppressed and a very strong nonlinear response can be observed at $B_1=0.9$. In figure 2 the AC response amplitude A of $\langle x(t)\rangle$ is shown as a function of B_1 at various frequencies for fixed value of $B_0=0.1$. The response amplitude A is seen to increase with increasing B_1 until reaching the crossover field B_1^* where the response

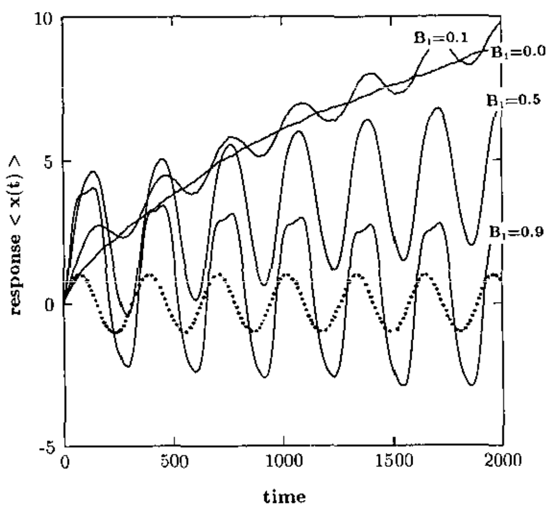

Figure 1. $\langle x(t)\rangle$ of a random walk particle in the external field of $B_0+B_1 \sin \omega t$ at selected values of B_1 with $B_0=0.1$ and $\omega=0.02$ fixed. The dotted line represents $\sin \omega t$.

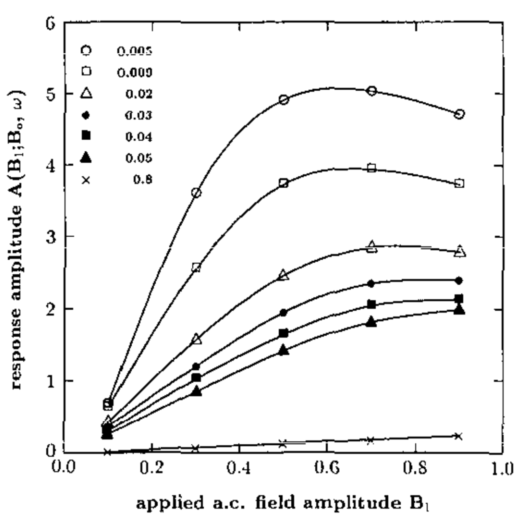

Figure 2. Plot of $A(B_1;\omega, B_0)$ as a function of B_1 for a constant bias field $B_0=0.1$ and various values of ω from 0.005 (○) to 0.8 (×).

amplitude A starts to decrease with further increasing B_1. This crossover field B_1^* of the maximum response amplitude seems to increase with increasing frequency and DC bias field B_0 although the overall amplitude diminishes with increasing frequency. At an extreme case of $\omega = 0.8$ the response amplitude A is seen to continue to increase very slowly as B_1 is increased with no sign of the crossover field B_1^* observed. This crossover field phenomenon of nonlinear response is derived from the competing interactions between drifting and trapping (Harder *et al* 1986, Havlin and Ben-Avraham 1987), both of which are affected by the DC bias field. When the frequency ω of the AC field increases the attempting frequency of escape from the dangling end traps is expected to increase. In figure 3 we have shown the phase shift $|\phi|$ of $\langle x(t)\rangle$ as a function of the AC field amplitude B_1 at various frequencies ω for a fixed DC bias field of $B_0 \doteq 0.1$. At lower frequencies the phase shift $|\phi|$ is observed to decrease rapidly with increasing field amplitude B_1. However, at higher frequencies the phase shift dependence on B_1 becomes much slower, and at an extreme of $\omega = 0.8$ we can see that the phase shift hardly depends on B_1 with convergence to $\phi \simeq k\pi$. In figure 4 the frequency dependence of the random walk response amplitude A is depicted for various values of B_1 and $B_0 = 0.1$. In the frequency range below $\omega = 0.05$ the random walk response amplitude A is seen to decrease with increasing frequency ω for all values of B_1. At each given frequency the response amplitude A can be seen to increase significantly with increasing field amplitude B_1 of the applied AC field at lower fields less than $B_1 = 0.5$ but does not depend very much on the field amplitude B_1 at higher fields above $B_1 = 0.5$. This nonlinear effect on the random walk response amplitude A at higher applied fields above $B_1 = 0.5$ conforms with the results of figure 1 where $\langle x(t)\rangle$ shows the nonlinear response at the AC field amplitude of $B_1 = 0.9$. In figure 5 the applied AC field (B_1) dependence of the random walk response amplitude (A) is shown for various DC bias fields (B_0) at a fixed AC frequency of $\omega = 0.02$. At lower DC bias fields less than $B_0 = 0.2$ we can see that the response amplitude A shows the crossover nonlinear behaviour with a large crossover field B_1^* at a higher DC bias field. The DC bias field tends to reduce the overall random walk response amplitude A. It

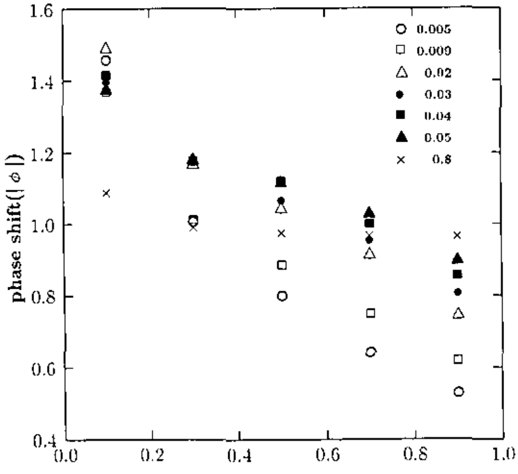

Figure 3. Plot of phase shift ($|\phi|$) as a function of B_1 for a constant bias field $B_0 = 0.1$ and various values of ω from 0.005 (○) to 0.8 (×).

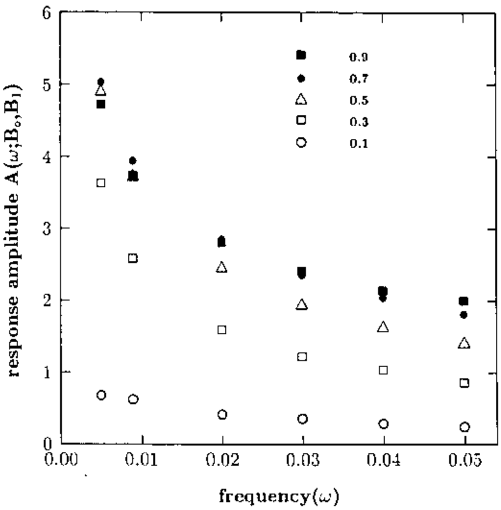

Figure 4. Plot of $A(\omega;\, B_0,\, B_1)$ as a function of ω for a constant bias field $B_0 = 0.1$ and various values of AC field amplitude B_1 from 0.1 (○) to 0.9 (■).

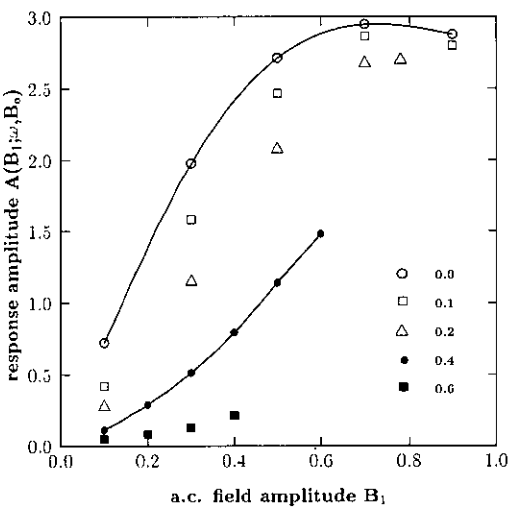

Figure 5. Plot of response amplitude $A(B_1;\, \omega,\, B_0)$ as a function of AC field amplitude B_1 for a constant frequency $\omega = 0.02$ and various values of DC bias field B_0 from 0.0 (○) to 0.6 (■). Note the boundary condition of $B_0 + B_1 = 1$.

can be seen also that at strong DC bias fields above $B_0 = 0.4$ the functional dependence of the response amplitude A on the AC field amplitude B_1 in the region below $B_1 = 0.4$ changes to be quadratic as compared with the linear dependence at smaller DC bias fields below $B_0 = 0.2$.

4. Conclusion

The DC bias field effect on the random walk response to the AC applied field in the percolation cluster may be classified into two characteristically different regimes with respect to the frequency ω and amplitude B_1 of the AC field. In the linear regime where the phase shift ϕ is negligible or converges to a constant value, the DC bias effect is equivalent respectively to an increase or a decrease of the AC field amplitude B_1. However, in the nonlinear effect regime where the phase shift ϕ strongly depends on the AC field amplitude (B_1), the DC bias field (B_0) effects become no longer equivalent to the simple enhancement or reduction of the AC field amplitude.

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